The Visual Estimation of Flow Rates of Aggregates and Slurries: Preliminary Studies

Harvey A. Cohen
Alan L. Harvey

Computer Science Department, La Trobe University
Bundoora, Melbourne, Victoria
AUSTRALIA 3083

The focus of this work has been to develop machine vision means for the estimation of the flow rates of slurries and aggregates such as occur in power houses, in mineral processing, and in the packaging and transfer of such materials. Our approach is to determine optic flow in orientation and magnitude from sequences of video images, either at frame-time delay or multiples thereof. In the materials whose flow was studied there is a definite texture not little in the way of defined lines. The actual velocity in cases of interest is primarily (if not exclusively) in one direction. In the preliminary study reported here we report on optical flow measurements performed on synthesised sequences of natural images. For the flow cases studied, it is shown that the "natural" estimate of optical flow velocity offers a better first estimate that the spatially smoothed gyrator of Horn and Schunk. A practical method of capturing image sequences using a frame grabber of single one capacity is explained.

Keywords: Optical flow, spatiotemporal filters, image sequences, image sequence capture, motion, slurries, tuned filters, three dimensional filters, image processing hardware.

Computation of Motion

There are two distinct approaches to the calculation of motion from sequences of images. [5] The first involves feature matching where the features involved may be lines or corners. In this method corresponding features on a sequence of image need to be matched in much the same way as in the matching of stereo pairs. The second method, pioneered primarily by Horn et al [6],[7] involves the computation of le optical flow as field of flow of grey scale within the "age sequence. The matching approach has obvious capability in the dynamic study of well-defined objects, such as machines and packages; whereas the global approaches have special significance for the study of the flow of textures aid slurries, and amorphous objects such as clouds. Hence in the work described here on the visual sensing of the flow of aggregates and slurries the approach via optic flow has been taken.

Optical flow is a local property related to the both the local gradient of grey scale in each image, and to the difference in grey scale at each pixel location in successive images. In the approach of Horn et al [6] the optical flow is essentially obtained by balancing the data values with smoothness constraint. Recently Heeger [1] demonstrated a simpler approach to optic flow estimation via the application of three dimensional convolution masks to image sequences. It is of interest to note that Heeger [1] proposed spatiotemporal filters on the basis of psychological models of human perception of motion. [cf Watson and Ahumada [2]. Adelson and Bergen [3]

Optic Flow

For a continuous image E(x,y) the optic flow equation is:

\[ u E_x(x,y,t) + v E_y(x,y,t) + \frac{\partial}{\partial x} E(x,y,t) = \frac{d}{dt} E(x,y,t) \]

where (u,v) is the flow velocity at (x,y). For rigid bodies

\[ \frac{d}{dt} E(x,y,t) = 0 \]

Some recent papers have discussed non-rigid motion, [1 l][12]. Our interest in slurries and aggregates ultimately requires the study of the non-zero (total) flow case. However, in our preliminary studies, we have used natural images, but then synthesised an image sequence. Hence in these preliminary studies we accept the rigid body condition, and impose the further restriction that the velocity flow component along the x-axis,

\[ u(x,y) = 0 \]
This leads to the simple formula:
\[
P(x,y,t) = \frac{\partial}{\partial t} \frac{\partial E(x,y,t)}{\partial x}
\]

In digital image processing optic flow thus depends on the numerical calculation of two partial derivatives. In numerical analysis, such a calculation is achieved by using either the forward difference operator:
\[
D_f(i) = f(i+1) - f(i)
\]
the backwards difference operator
\[
D_{backward}(i) = f(i) - f(i-1)
\]
or a central difference operator
\[
D_{central}(i) = \frac{f(i+1) - f(i-1)}{2}
\]

It is notable that these, and other, difference operators take the form
\[
D^*(S*f)
\]
where the forwards difference operator is applied to a convolved image, or equivalently
\[
S^*(D*f)
\]
where a convolution operator is applied after the forwards operator. (The form of S is trivially different in the last two formulas). In the case of backwards difference operator S is merely a translation operator, but otherwise is a smoothing operator.

\[
D_{central} = \frac{1}{2} (1 1) * D.
\]

More generally, restricting attention to decomposable operators, a general form for the smoothed numerical derivative in the y direction is
\[
D_y (smoothed) = S_x S_y S_t * (D_y)
\]
\[
D_t (smoothed) = S_x S_y S_t * (D_t)
\]

In their pioneering paper on optical flow, Horn and Schunk used the 1-D smoothing convolutions to calculate a smoothed derivative \( E_{ij} [H & S] \):
\[
S_x = \frac{1}{2} (1 1); \quad S_y = (1) \quad S_t = \frac{1}{2} (1 1)
\]

In detail, the smoothed partial derivative used by Horn and Schunk yield a smoothed partial derivative \( E_{ij} [H & S] \):
\[
\frac{1}{4} \left( E(i,j+1,t)+E(i,j+1,t+1)+E(i+1,j+1,t)+E(i+1,j+1,t+1) \right)
\]
\[
- \frac{1}{4} \left( E(i,j,k)+E(i,j,k+1)+E(i+1,j,k)+E(i+1,j,k+1) \right)
\]

Similarly Horn and Schunk smooth out the numerical partial temporal derivative:
\[
S_x = \frac{1}{2} (1 1); \quad S_y = \frac{1}{2} (1 1); \quad S_t = (1)
\]

yielding as a smoothed time derivative \( E_{ij} [H & S] = \)
\[
\frac{1}{4} \left( E(i,j+1,t)+E(i,j+1,t+1)+E(i+1,j+1,t)+E(i+1,j+1,t+1) \right)
\]
\[
- \frac{1}{4} \left( E(i,j,k)+E(i,j,k+1)+E(i+1,j,k)+E(i+1,j,k+1) \right)
\]

In this paper we refer to the unsmoothed estimate of the velocity component \( v \) (in the case where by construction \( u=0 \)) as
\[
v [H & S] = \frac{E_{ij} [H & S]}{E_{ij} [H & S]}
\]

where as above the D operators are the usual forwards difference operators, the convolving vectors (1, 1). Likewise what Horn and Schunk describe as involving “consistent estimates” of the partial derivatives is the velocity estimate:
\[
v [H & S] = \frac{E_{ij} [H & S]}{E_{ij} [H & S]}
\]

Put another way, simple estimate for \( v \) used in this paper involves two spatiotemporal filters both 1*1*1*2 whereas Horn and Schunk use smoothed spatial-temporal filters that are 2*2*2*2 in size. The point that must be stated is that the use of spatiotemporal filters in this arena is inevitable: however, some workers have experimented with spatiotemporal filters of wider support. [1],[3].

Image Sequence Capture
Recently commercial frame grabbers have been released which have a capability for capturing and digitizing a sequence of image frames. The work described here was done without such an advanced frame grabber, and employed a first generation frame grabber (by Imaging Technology Incorporated) with single frame capture capability. The captured frame is interfaced, and thus features two images with time disparity of (1/25) second. Thus the formulas introduced above involving just an image pair can be used, provided one bears in mind the missing rows in the interlaced image pair. Work to this effect is in progress now.
Fig 1 The first image of a two image sequence displaying sub-pixel flow per frame. This image is derived from the upper section of Brodatz image D54 "Pebbles" and is reasonably akin to aggregates of interest. The second image of the frame differs negligibly to the eye, due to the sub-pixel alterations, which were determined by linear interpolation of pixel values.

**Experimental Study**

The results reported herein utilise two image sequences, of which the first is based on a Brodatz texture number 54, Pebbles, from the Brodatz compilation [8]; the second is displaced by a velocity field in the y direction. In particular: Case A: Uniform velocity used to generate second image.

Case B: Velocity uniform increases 0 - max - 0.

The actual image is 512 pixels wide, but only central 500 pixels utilised. in H&C calculation, 501 for H&S.

The velocity field has been computed from the two image pairs

(i) Using Horn Schunck formula

(ii) Using direct formula.

Typical results are presented in Fig 2.

**Conclusions**

This study was based on the needs for a robust means of determining visually the predominantly unidirectional flow rates of aggregates and slurries. In practical situations, the camera will be placed so as to examine a section of approximately uniform motion, modeled by our Case A, or will be set to view flow through a sluice or pipe where edge velocity is zero, and maximum velocity is in the central region, as modelled by our Case B. The graphs presented show that the Horn and Schunk expression is a poor estimate for flow velocity. However the direct expression we have used gives a good estimate of flow velocity where it can be applied; where the temporal gradient at a pixel location is zero this quantity cannot be determined.

**References**


Fig (2): Comparison of two direct measures of optical flow deduced from two image sequence of "Pebbles" of Fig (1).

In examples on the left the synthesised image movement is 0.7 pixels:

\[ v(\text{known}) = 0.7 \text{ pixels/ frame} \]

In examples on the right the second image was synthesised

\[ v(\text{known}) = \begin{cases} \frac{x}{250} & 0 \leq x \leq 250 \\ \frac{500-x}{250} & 250 \leq x \leq 500 \end{cases} \]

In the top two plots, the Horn and Schunck first estimate of velocity for this unidirectional flow is given, using the formula:

\[ v_{(H \& S)} = \frac{E_x(i,j)(H \& S)}{E_y(i,j)(H \& S)} \]

In the bottom two plots the simpler first estimate

\[ v_{(C \& H)} = \frac{D_x E(i,j,t)}{D_y E(i,j,t)} \]

In all plots the known velocity is shown.

Note that Horn and Schunk use the above velocity as only the initial estimate for an iterative smoothing procedure.